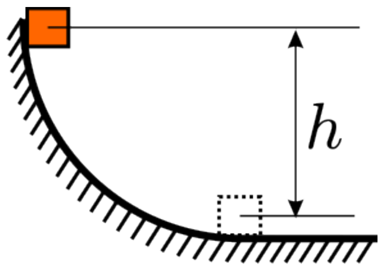
WORK AND KINETIC ENERGY

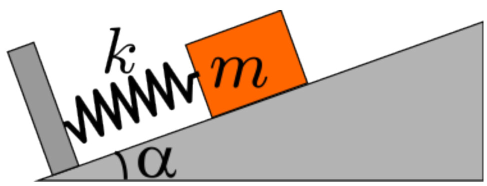
1. **Varying force:** The force F acting on the box is given as $F = (\alpha x - \beta x^2)\,\hat{\imath} + (-\gamma y^3)\,\hat{\jmath}$, where α, β, γ are known constants and $\vec{r} = x\hat{\imath} + y\hat{\jmath}$ is displacement in the direction of motion. Assuming that F is the only force acting on the box find work done on the box when the box is displaced by the force from position $(0,0)$ to position (x_0, y_0). Express you answer in terms of $g, \alpha, \beta, \gamma, x_0$ (not all may be necessary).

2. **Box on hemisphere**. Box with mass m is placed on rough surface. The surface has shape of ¼ of the cylinder as it is shown on the figure. The radius of the cylinder is known and equal h. Once released, the box slides down and stops because of friction right at the bottom of the cylinder. Find work done by friction during this motion. Express you answer in terms of g, h, m (not all may be necessary)



3. **Box with friction**. A box of mass m has speed of v_0 on rough horizontal surface at moment of time $t = 0$. Coefficient of kinetic friction between the box and the surface is μ. Find the distance box will travel from moment of time $t = 0$ till complete stop. Express you answer in terms of g, m, v_0, μ (not all may be necessary).

4. **Spring.** Box of mass m in pressed against a spring on frictionless incline. Incline making angle of α with respect to the horizontal surface. The spring constant of the spring is k. Spring is initially compressed on Δx. Answer question below in terms of $g, \alpha, m, k, \Delta x$ (not all may be necessary).



   (a) The box was released at $t = 0$. Find speed of the box at the moment when the spring is completely decompressed

   (b) Find Kinetic energy of the box at the top of its trajectory

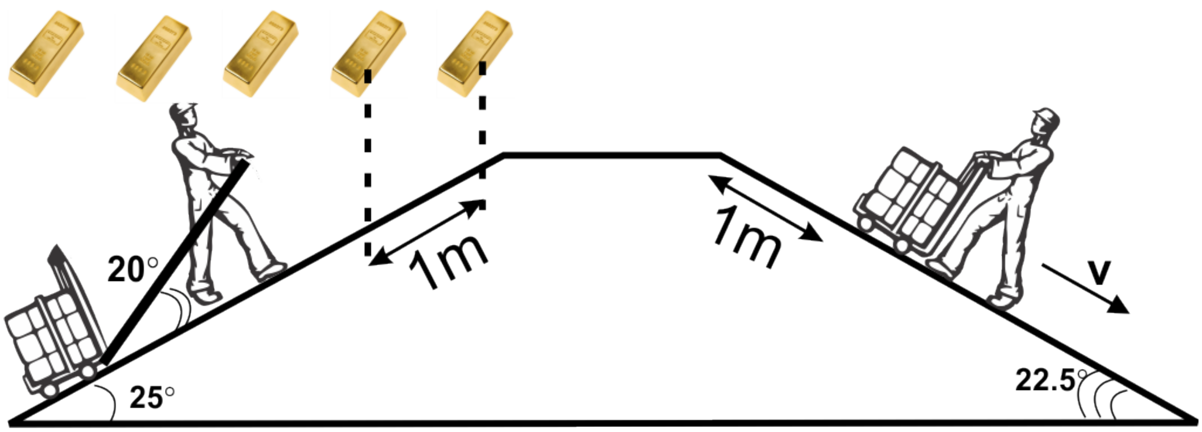
5. **Physics Professor**: A physics professor is pushed up a ramp inclined upward at 30.0° above the horizontal as she sits in her desk chair, which slides on frictionless rollers. The combined mass of the professor and chair is M. She is pushed L along the incline by a group of students who together exert a constant horizontal force of F. The professor's speed at the bottom of the ramp is v_0. Use the work-energy theorem to find her speed at the top of the ramp.

6. **Kinetic Sculpture**: Part of artist's sculpture consists of a object with mass M (you can't tell what it is supposed to be, but it's art) and another object with mass m ($m < M$) which hang straight down from opposite ends of a very thin, flexible wire. This wire passes over a smooth, cylindrical, horizontal, stainless steel pipe H above the floor. The frictional force between the rod and the wire is negligible. The M object is held h_1 above the floor and the other object hangs h_2 above the floor ($h_2 < h_1$). When the mechanism releases the M object, both objects accelerate and one will eventually hit the floor -- but they don't hit each other. Calculate the speed of the object which hits the floor.

7. **Ice Storm**: The safety system to protect drivers going down hills during an ice storm consists of a bumper, which can be considered a stiff spring, at the bottom of the hill. In the scenario you are given, the car, mass m, starts from rest at the top of a hill which makes an angle θ with the horizontal. The distance that the car slides from the top of the hill until it is stopped by the spring is L. For the worst case scenario, assume that there is no frictional force between the car and road due to the ice. If the maximum compression of the spring from its equilibrium position is D, your job is to calculate the required spring constant k in terms of m, D, L, and θ.

8. **Physics Student**: A physics student spends part of her day walking between classes or for recreation, during which time she expends energy at an average rate of 300 W. The remainder of the day she is sitting in class, studying or resting; during these activities, she expends energy at an average rate of 100 W. If she expends a total of 1 x 10^7 J of energy in a 24 hour day, how much of the day did she spend walking?

9. **Worker.** A worker pulls a cart with mass of $20\ kg$ uphill using rope with constant force of $P = 300\ N$ as it is shown in the figure. The coefficient of rolling friction between the cart and the surface is $\mu_r = 0.01$. At the moment $t = 0$, an automatic crane adds 1 gold bar with mass of 10 kg. The crane adds an additional bar after each meter traveled until a total of 5 bars are loaded. The worker then passes over the top of the ramp and begins to descend down the other side. In order to maintain a constant speed on the descent, the worker pushes against the cart.



(a) What work was done by the worker between the first and the last load of the golden bars?

(b) What direction should he apply a force on 22.5-degree slope to slow down the cart most efficiently?

(c) What should be the minimal mass of the worker if the coefficient of the kinetic friction between his boots and the slope is $\mu_k = 0.5$.

(d) Assuming the worker stops the cart at the top of the hill before his descent, what is the speed of the worker on this way down assuming that he first let cart to accelerate for 1 meter before applying his maximum force?