Physics 218 – Comprehensive Exam

Fall 2017 (all sections) **December 1st, 2017**

Please fill out the information and read the instructions below, but **do not open the exam** until told to do so.

Rules of the exam:

1. You have 120 minutes (2 hrs) to complete the exam.
2. Formulae are provided to you with the exam on a separate sheet. Make sure you have one before the exam starts. You may *not* use any other formula sheet.
3. Check to see that there are 10 numbered (5 double-sided) pages in addition to the scantron-like cover page. **Do not remove any pages**.
4. If you run out of space for a given problem, the last page has been left blank and may be used for extra space. Be sure to indicate *at the problem under consideration* that the extra space is being utilized so the graders know to look at it!
5. Calculators of any type are **not allowed**. Ensure that all your answers are in terms of the known variables given in the question.
6. Cell phone use during the exam is strictly prohibited. Please turn off all ringers as calls during an exam can be quite distracting.
7. Be sure to put a box around your final answer(s) and clearly indicate your work. Credit can be given **only** if your work is legible, clearly explained, and labelled.
8. All of the questions require you show your work and reasoning.
9. Have your TAMU ID ready when submitting your exam to the proctor.

Fill out the information below and sign to indicate your understanding of the above rules

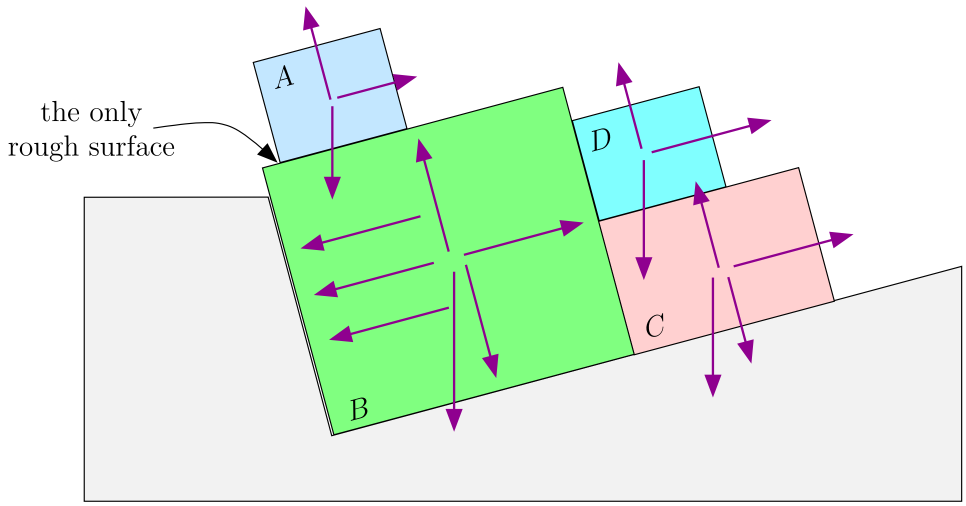
Name: (printed *legibly*) ______________________________ UIN: ______________________________

Signature: ______________________________ Section Number: ____________________

Instructor (circle one): Akimov Dierker Melconian

Short Problems:

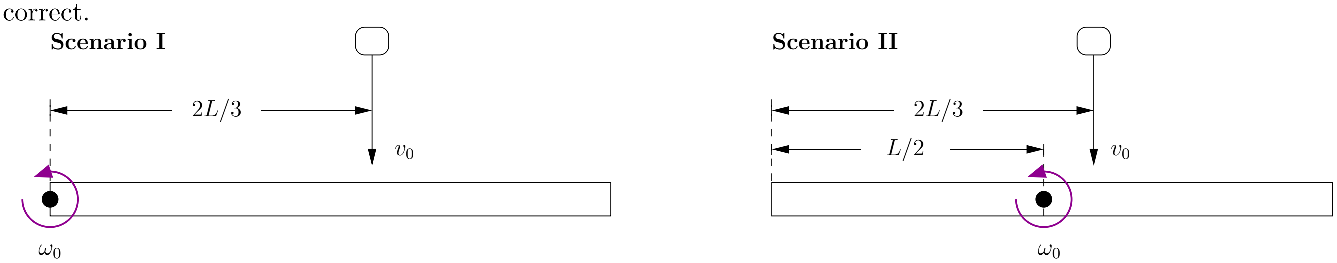
A) Four boxes, A, B, C and D, are in static equilibrium as shown. All surfaces are frictionless except between boxes A and B where the surfaces are rough. Shown are arrows for all forces acting on these blocks (not to scale). Clearly label all the forces shown which are part of an action-reaction pair (do not include the Earth or the inclined surface). Identify the pairs by circling your labels and connecting them with a line.



LO	S	U
22.1		
22.2		
22.3		
22.4		
22.5		

B) A point-like object of mass m is moving horizontally with a speed v_0 as a rod of length L and mass M is rotating counter-clockwise with an angular velocity ω_0.

(a) For each of the two scenarios shown (as viewed from above the horizontal plane), calculate the value of the angular momentum of each of the two objects about the rotation axis indicated, ensuring their relative signs are correct.



(b) When the mass m collides with the rod in Scenario I, it explodes and its pieces go flying in various directions.

i. Is the total linear momentum conserved before and after the collision/explosion? *Briefly* explain your answer.

LO	S	U
57.1		
57.2		
57.3		
57.4		
48.1		
59.1		

ii. Is the total angular momentum conserved before and after the collision/explosion? *Briefly* explain your answer.

C) A startled armadillo leaps vertically upward in the $+\hat{\jmath}$ direction with a speed v_0. It rises to a height h in a time t.

(a) What is its speed, v_h, at the height h? Express your answer in terms of h, v_0 and g.

(b) What was its average velocity, $\langle \vec{v} \rangle$, from when it started the jump to when it first reached the height, h?

LO	S	U
3.1		
14.1		
11.1		

D) For each of the following, solve for the unknown:

(a) When solving a projectile motion problem, you end up with the equation $y = y_0 + v_0 t - \frac{1}{2} g t^2$. What is t in terms of the other variables?

(b) Given $x + 5y = -13$ and $2x - y = 7$, what are x and y?

(c) Given $\dfrac{1}{\alpha + \beta} = \dfrac{-1}{4}$ and $\dfrac{\alpha + \beta}{\alpha - \beta} = \dfrac{-2}{5}$, what are α and β?

LO	S	U
3.2		
5.1		
4.1		
4.2		

E) Unit conversion. For the following, you may leave your answers in terms of fractions and/or square roots:

(a) The kinetic energy of an object is $K = 1$ J. What is K in units of $\mathrm{kg\,cm^2/min^2}$?

(b) The potential energy of an object is $U = 1\ \frac{\mathrm{kN}}{\mathrm{cm}}\mathrm{mm}^2$. What is U in units of J?

(c) The period of an object is $T = 10\ \sqrt{\frac{\mathrm{kg\,cm}}{\mathrm{N}}}$. What is the period in seconds?

LO	S	U
10.1		
10.2		
10.3		

F) A comet is in an elliptical orbit about the Sun.

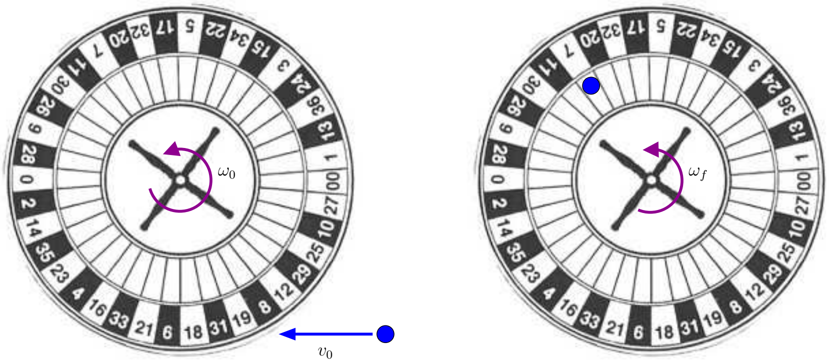
(a) Is the maximum speed at the perihelion or the aphelion (see the figure in the bottom-right portion of the formula sheet)? *Briefly* explain your choice.

(b) What is the distance from the centre of the Sun to the perihelion, in terms of the semi-major axis, a, and eccentricity, e?

LO	S	U
63.1		
64.1		

Prob 1 A roulette wheel may be approximated as the assembly of a uniform solid disk of radius R and mass $200M$ with four uniform rods each of mass $27M$ which span the length from the centre of the wheel out to $R/3$. A point-like ball of mass $16M$ is thrown with a speed $v_0 = 2\omega_0 R$ at the very edge of the wheel as shown on the left. After it has spun and bounced around, it eventually settles in the lucky #7 slot as shown, a distance $3R/4$ from the centre. Assume that friction (which will eventually bring the roulette wheel to rest) is negligible over the time it takes for the ball to fall in the #7 slot.

(a) Determine the total moment of inertia of the roulette wheel with the axis of rotation through it's centre as shown.



(b) What is the final angular speed of the system? Your answer should be a fraction of two integers times the initial angular speed of the roulette wheel.

(c) What are the initial <u>and</u> final kinetic energies of the system, in terms of $MR^2\omega_0^2$?

(d) Is the collision of the ball with the roulette wheel elastic or inelastic? *Briefly* explain your answer.

LO	S	U
51.1		
51.2		
53.1		
3.3		
57.5		
57.6		
59.2		
34.1		
35.1		
35.2		
50.1		

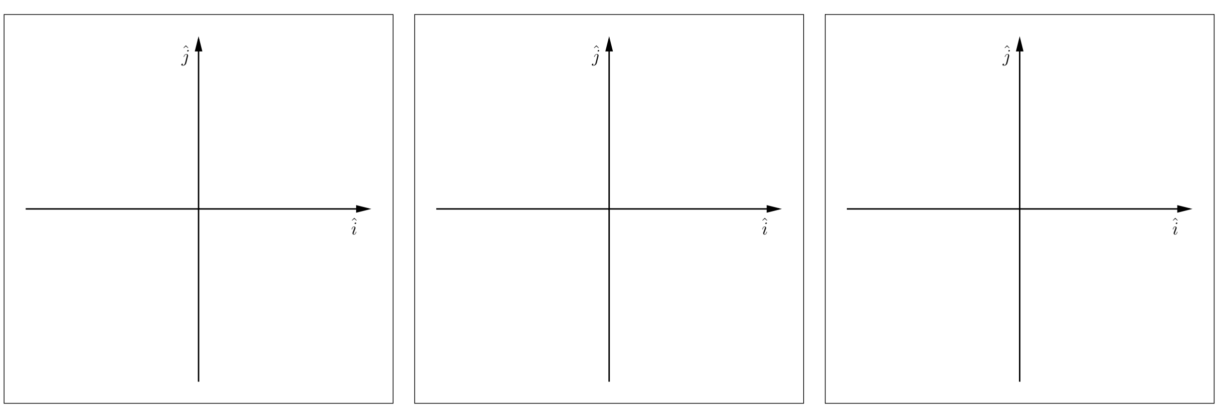
Prob 2 A rifle is aimed horizontally in the $+\hat{\imath}$ direction at the center of a target a distance d away.

(a) What is the bullet's speed, $v(t_0)$, when it strikes the target at time t_0? Express your answer in terms of d, t_0 and g.

(b) In order to hit the center of the target, another bullet is fired at an angle θ above the horizontal x-axis. The time it takes to reach the $\hat{x}$-position of the target is the same t_0 as above (the bullet is shot a little faster). What is this angle?

LO	S	U
1.1		
11.2		
14.2		
1.2		
3.4		
11.3		
14.3		
13.1		
13.2		
13.3		
13.4		
13.5		
13.6		

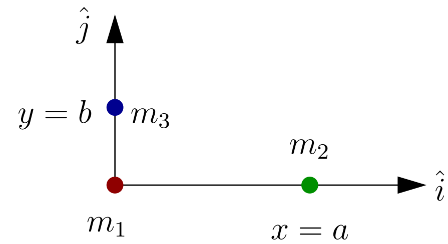
(c) For the trajectory in part (b), sketch on the following graphs the direction of the velocity and acceleration vectors for the bullet at times $\frac{1}{4}t_0$, $\frac{1}{2}t_0$ and $\frac{3}{4}t_0$. (The position of the bullet is at the local origin of each of the graphs).



$t = \frac{1}{4}t_0$ $t = \frac{1}{2}t_0$ $t = \frac{3}{4}t_0$

Prob 3 A system far away from any other objects consists of three particles at rest with masses m_1, m_2 and m_3. Mass m_1 is located at the origin, m_2 is at $x = a\hat{i}$ and m_3 is at $y = b\hat{j}$, as shown in the figure.

(a) What is the resultant gravitational force acting on mass m_1?



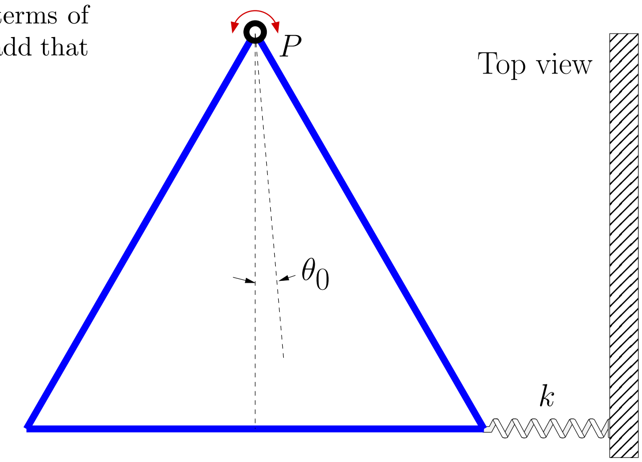
(b) What is the total potential energy of the system?

(c) How much work is required to move the mass m_2 from where it is initially to $x = \infty$? Take the mass to be at rest initially (at $x = a$) as well as when it is moved to $x = \infty$.

LO	S	U
60.1		
60.2		
60.3		
61.1		
61.2		
61.3		
61.4		
61.5		
39.1		
61.6		

Prob 4 A pendulum is constructed with three rods, each of length L and mass M, forming a rigid equilateral triangle as shown below. The top of the triangle has an axis through it, enabling frictionless rotation around that point, P. The pendulum is placed on a horizontal frictionless surface and connected to the wall with spring of force constant k as shown. The pendulum is displaced by a small angle, θ_0, from equilibrium and then released. Note that the moment of inertia for the bottom rod about the axis of rotation shown is $I_{\text{bot rod}} = \frac{5}{6}ML^2$.

(a) Find the total moment of inertia, I_P, of the pendulum in terms of M and L. (*Hint:* Find I for the two top rods about P and add that to $I_{\text{bot rod}}$ for the 3rd rod given above).



(b) Prove that the pendulum would undergo simple harmonic motion. You may assume $\theta_0 \ll 1$ and that the force of the spring is approximately horizontal. Also, note that $\cos(60°) = \sin(30°) = \frac{1}{2}$ and $\sin(60°) = \cos(30°) = \frac{\sqrt{3}}{2}$.

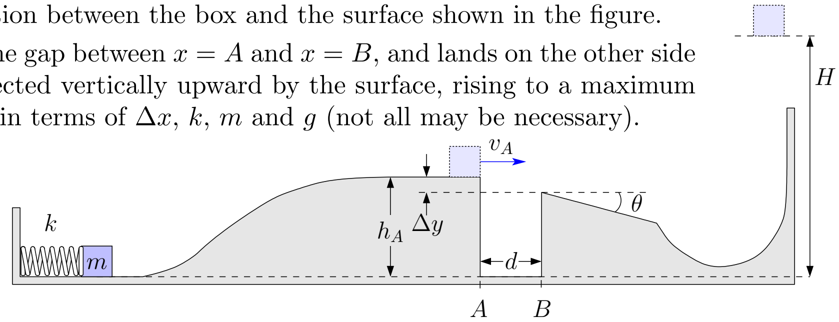
(c) What is the angular frequency, ω, of pendulum's motion? Express your answer in terms of I_P, k, M and L (not all may be necessary).

(d) Find the maximum angular velocity, $\Omega = d\theta/dt$, of the pendulum. Express your answer in terms of I_P, k, M, θ_0 and L (not all may be necessary).

LO	S	U
51.3		
53.2		
1.3		
55.1		
54.1		
66.1		
66.2		
3.5		
66.3		

Prob 5 A box of mass m is pressed against a strong spring which has a force constant k. The spring is compressed a distance Δx and then released from rest. There is no friction between the box and the surface shown in the figure.

(a) Assume that the box successfully flies over the gap between $x = A$ and $x = B$, and lands on the other side without any loss of energy. It then gets directed vertically upward by the surface, rising to a maximum height, H. Find H, expressing your answer in terms of Δx, k, m and g (not all may be necessary).



(b) What will be the kinetic energy at point A if the height is h_A above its starting point? Express your answer in terms of $\Delta x, k, m, h_A$ and g (not all may be necessary).

(c) When the box lands at point B, its velocity is in the same direction as the incline, which has an angle θ with respect to the horizontal. What should $\cos\theta$ be if the gap between points A and B has a height Δy and width d. Take the speed at point A to be v_A, and express your answer in terms of m, Δy, d, v_A and g (not all may be necessary).

LO	S	U
3.6		
38.1		
38.2		
40.1		
38.3		
38.4		
40.2		
1.4		
4.3		
34.2		
38.5		
40.3		

Extra Space: