

The derivative can be defined in terms of a limit. For a function $f(t)$ that depends on the variable t , the derivative is $f'(t) = \frac{df}{dt} = \lim_{\Delta t \rightarrow 0} \frac{\Delta f}{\Delta t}$. This corresponds to finding the tangent to the curve at the point t . In practice, for a polynomial, the derivative can be found using the following formula: if $f(t) = a_{-2}t^{-2} + a_{-1}t^{-1} + a_0t^0 + a_1t^1 + a_2t^2 + a_3t^3 + \dots + a_nt^n$ where the a_i and n are constants, then $f'(t) = \frac{-2a_{-2}}{t^3} - \frac{a_{-1}}{t^2} + 0 + a_1 + 2a_2t + 3a_3t^2 + \dots + na_nt^{n-1}$. Recall that $1/t^n = t^{-n}$, $t^0 = 1$ and $t^1 = t$.

The “chain rule” says that if a function f depends on g which itself depends on the variable x , *i.e.* $f(g(x))$, then the derivative of f with respect to x is $\frac{df}{dx} = \left(\frac{df}{dg}\right)\left(\frac{dg}{dx}\right)$. For example, if $f = 2g^2$ and $g = x + 3x^2$, then $df/dx = (4g)\frac{dg}{dx} = (4[x + 3x^2])(1 + 6x)$.

1. Find the derivative of $y = 5 + 6x$
2. Find the derivative of $f(t) = 10t^{10}$
3. Find the derivative of $f(x) = 3x^3 - 9x + 1$
4. Find the derivative of $y = 3t^3 - 12t^2 + 23t$
5. Find the derivative of $f(t) = 2t^9 - 5t^{-9} + 9t$
6. Find the derivative of $z = 2y^{-6} - 4y^{-4} + 6y^{-2} + 8$
7. Find the derivative of $y = 2\sqrt{x} + 3\sqrt[3]{x} - 4\sqrt[4]{x}$
8. Find the derivative of $f(x) = 2x^{3/5} - 4x^{7/4} + 3x^{8/3} - 8$
9. Find the derivative of $f(t) = \frac{1}{t} - \frac{1}{t^3} + \frac{1}{t^5}$
10. Find the derivative of $g(z) = \frac{2}{z^3} \left(1 + \frac{1}{2z^2} - \frac{3}{z^4}\right)$
11. Find the derivative of $y = x^2(5x^2 - 2)$
12. Find the derivative of $y = (2t - 3)(3t + 2t^2)$
13. Find the derivative of $f(x) = \frac{4 - 7x + 8x^3}{x}$
14. Find the derivative of $r(t) = \frac{5t^5 - t^3 + 4t}{t^3}$
15. Find where (if anywhere) the function $v(t) = \frac{1}{3}t^3 + t^2 - 15t + 2200$ isn't instantaneously changing
16. Find where (if anywhere) the function $a(t) = t^5 - 2t^4 - 5t^3$ isn't instantaneously changing
17. Determine where the function $f(x) = 600 - 40x^3 - 5x^4 + 4x^5$ is increasing and decreasing.
18. Determine where the function $f(x) = (x + 3)(x - 1)^2$ is increasing and decreasing.
19. Determine where, if anywhere, the tangent line to $f(x) = \frac{1}{3}x^3 - x^2 + 3x$ is parallel to the line $y = 2x + \frac{1}{2}$.
20. Determine where, if anywhere, the tangent line to $v(t) = 3/t - t/3$ is parallel to the line $\omega(t) = 9 - t/2$