

Phys 206 — Exam III Formulae

Trigonometry and Vectors:

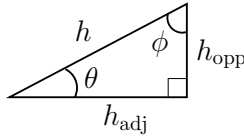
$$\sin 30^\circ = \cos 60^\circ = \frac{1}{2} \quad \sin 36.9^\circ \approx \cos 53.1^\circ \approx \frac{3}{5}$$

$$\sin 45^\circ = \cos 45^\circ = \frac{1}{\sqrt{2}} \quad \sin 53.1^\circ \approx \cos 36.9^\circ \approx \frac{4}{5}$$

$$\sin 60^\circ = \cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$h_{\text{adj}} = h \cos \theta = h \sin \phi$$

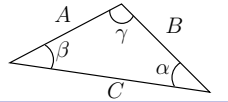
$$h^2 = h_{\text{adj}}^2 + h_{\text{opp}}^2$$



$$h_{\text{opp}} = h \sin \theta = h \cos \phi$$

$$\tan \theta = \frac{h_{\text{opp}}}{h_{\text{adj}}}$$

Law of cosines: $C^2 = A^2 + B^2 - 2AB \cos \gamma$



Law of sines: $\frac{\sin \alpha}{A} = \frac{\sin \beta}{B} = \frac{\sin \gamma}{C}$

$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k} \quad \hat{A} = \frac{\vec{A}}{|\vec{A}|}$$

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z = AB \cos \theta = A_{\parallel} B = AB_{\parallel}$$

$$\vec{A} \times \vec{B} = (A_y B_z - A_z B_y) \hat{i} + (A_z B_x - A_x B_z) \hat{j} + (A_x B_y - A_y B_x) \hat{k}$$

$$|\vec{A} \times \vec{B}| = AB \sin \theta = A_{\perp} B = AB_{\perp} \quad (\text{direction via right-hand rule})$$

Quadratic:

$$ax^2 + bx + c = 0 \Rightarrow x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Derivatives:

$$\frac{d}{dt}(at^n) = nat^{n-1}$$

$$\frac{d}{dt} \sin at = a \cos at$$

$$\frac{d}{dt} \cos at = -a \sin at$$

Integrals:

if $f(t) = at^n$, then $\begin{cases} \int_{t_1}^{t_2} f(t) dt = \frac{a}{n+1} (t_2^{n+1} - t_1^{n+1}) \\ \int f(t) dt = \frac{a}{n+1} t^{n+1} + C \end{cases}$
($n \neq -1$)

$$\int \sin at dt = -\frac{1}{a} \cos at$$

$$\int \cos at dt = \frac{1}{a} \sin at$$

Constants/Conversions:

$$g = 9.80 \text{ m/s}^2 = 32.15 \text{ ft/s}^2 \quad (\text{Earth, sea level})$$

$$\approx 10 \text{ m/s}^2 \approx 33 \text{ ft/s}^2$$

$$1 \text{ mi} = 1609 \text{ m}$$

$$1 \text{ lb} = 4.448 \text{ N}$$

$$1 \text{ ft} = 12 \text{ in}$$

$$\Leftrightarrow 0.454 \text{ kg (Earth, sea level)} \quad 1 \text{ in} = 2.54 \text{ cm}$$

$$1 \text{ rev} = 360^\circ = 2\pi \text{ radians}$$

Circular motion:

$$a_{\text{rad}} = \frac{v^2}{R} \quad a_{\text{tan}} = \frac{d|\vec{v}|}{dt} = R\alpha$$

$$T = \frac{2\pi R}{v} \quad s = R\theta \quad v_{\text{tan}} = R\omega$$

Relative velocity:

$$\vec{v}_{A/C} = \vec{v}_{A/B} + \vec{v}_{B/C}$$

$$\vec{v}_{A/B} = -\vec{v}_{B/A}$$

Forces:

Newton's Law: $\sum \vec{F} = m\vec{a}$, $\vec{F}_{B \text{ on } A} = -\vec{F}_{A \text{ on } B}$

Hooke's Law: $F_x = -k\Delta x$

friction: $|\vec{f}_s| \leq \mu_s |\vec{n}|$, $|\vec{f}_k| = \mu_k |\vec{n}|$

Kinematics:

translational

$$\langle \vec{v} \rangle = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1} \quad \vec{v} = \frac{d\vec{r}}{dt}$$

$$\langle \vec{a} \rangle = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} \quad \vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$$

$$\vec{r}(t) = \vec{r}_0 + \int_0^t \vec{v}(t') dt'$$

$$\vec{v}(t) = \vec{v}_0 + \int_0^t \vec{a}(t') dt'$$

— constant (linear/angular) acceleration only —

$$\vec{r}(t) = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2} \vec{a} t^2$$

$$\vec{v}(t) = \vec{v}_0 + \vec{a} t$$

$$v_x^2 = v_{x,0}^2 + 2a_x(x - x_0)$$

(and similarly for y and z)

$$\vec{r}(t) = \vec{r}_0 + \frac{1}{2}(\vec{v}_i + \vec{v}_f)t$$

rotational

$$\langle \omega \rangle = \frac{\theta_2 - \theta_1}{t_2 - t_1} \quad \omega = \frac{d\theta}{dt}$$

$$\langle \alpha \rangle = \frac{\omega_2 - \omega_1}{t_2 - t_1} \quad \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

$$\theta(t) = \theta_0 + \int_0^t \omega(t') dt'$$

$$\omega(t) = \omega_0 + \int_0^t \alpha(t') dt'$$

$$\theta(t) = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$$

$$\omega(t) = \omega_0 + \alpha t$$

$$\omega_f^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

$$\theta(t) = \theta_0 + \frac{1}{2}(\omega_i + \omega_f)t$$

Energy and Momenta:

translational

$$K = \frac{1}{2} M v^2$$

$$W = \int \vec{F} \cdot d\vec{r} \xrightarrow{\text{const force}} \vec{F} \cdot \Delta\vec{r}$$

$$P = \frac{dW}{dt} = \vec{F} \cdot \vec{v}$$

$$\vec{p}_{\text{cm}} = m_1 \vec{v}_1 + m_2 \vec{v}_2 + \dots$$

$$= M \vec{v}_{\text{cm}}$$

$$\vec{J} = \int \vec{F} dt = \Delta\vec{p}$$

$$\sum \vec{F}_{\text{ext}} = M \vec{a}_{\text{cm}} = \frac{d\vec{p}_{\text{cm}}}{dt}$$

$$\sum \vec{F}_{\text{int}} = 0$$

if $\sum F_{\text{ext},x} = 0$, $p_{\text{cm},x} = \text{const}$ if $\sum \tau_{\text{ext},z} = 0$, $L_z = \text{const}$

— Work-energy and potential energy —

$$W = \Delta K \quad W = -\Delta U \quad E_{\text{tot},i} + W_{\text{other}} = E_{\text{tot},f}$$

$$U(r) = -\int \vec{F} \cdot d\vec{r}; \quad U_{\text{grav}} = Mgy_{\text{cm}}; \quad U_{\text{elas}} = \frac{1}{2} k \Delta x^2$$

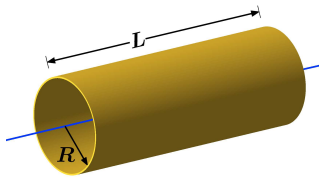
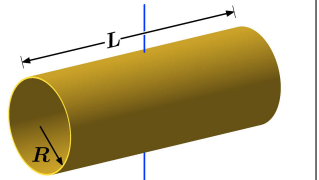
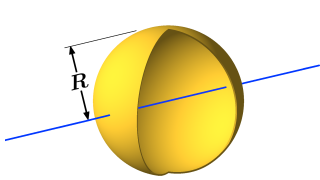
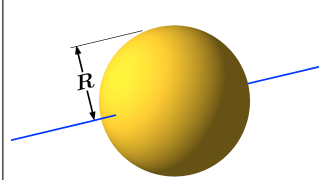
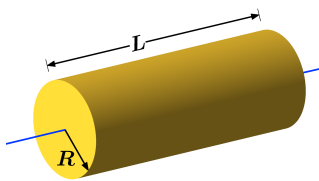
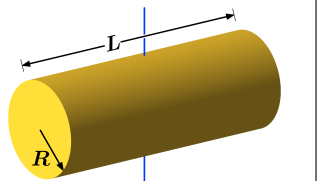
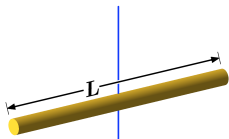
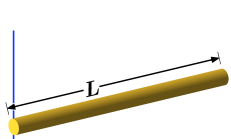
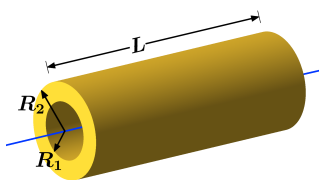
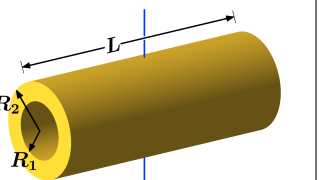
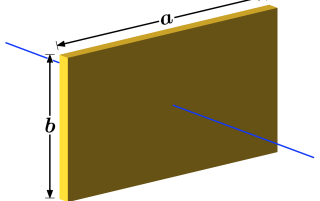
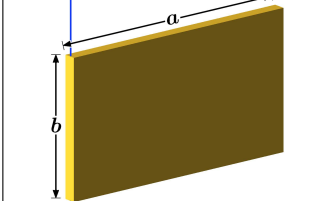
$$F_x(x) = -dU(x)/dx \quad \vec{F} = -\vec{\nabla}U = -\left[\frac{\partial U}{\partial x} \hat{i} + \frac{\partial U}{\partial y} \hat{j} + \frac{\partial U}{\partial z} \hat{k}\right]$$

Centre-of-mass:

$$\vec{r}_{\text{cm}} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2 + \dots + m_n \vec{r}_n}{m_1 + m_2 + \dots + m_n}$$

(and similarly for \vec{v} and \vec{a})

Moments of inertia:

<p>Thin cylinder</p>  <p>$I = MR^2$</p>	<p>Thin cylinder</p>  <p>$I = \frac{1}{2}MR^2 + \frac{1}{12}ML^2$</p>	<p>Hollow sphere</p>  <p>$I = \frac{2}{3}MR^2$</p>	<p>Solid sphere</p>  <p>$I = \frac{2}{5}MR^2$</p>
<p>Solid cylinder</p>  <p>$I = \frac{1}{2}MR^2$</p>	<p>Solid cylinder</p>  <p>$I = \frac{1}{4}MR^2 + \frac{1}{12}ML^2$</p>	<p>Thin rod</p>  <p>$I = \frac{1}{12}ML^2$</p>	<p>Thin rod</p>  <p>$I = \frac{1}{3}ML^2$</p>
<p>Thick cylinder</p>  <p>$I = \frac{1}{2}M(R_1^2 + R_2^2)$</p>	<p>Thick cylinder</p>  <p>$I = \frac{1}{4}M(R_1^2 + R_2^2) + \frac{1}{12}ML^2$</p>	<p>Rectangular plate</p>  <p>$I = \frac{1}{12}M(a^2 + b^2)$</p>	<p>Rectangular plate</p>  <p>$I = \frac{1}{3}Ma^2$</p>

↪ For a point-like particle of mass M a distance R from the axis of rotation: $I = MR^2$

↪ Parallel axis theorem: $I_p = I_{cm} + Md^2$

↪ A disk is a cylinder of negligible length; the I for a disk may be found by setting $L = 0$ in the formulae for cylinders