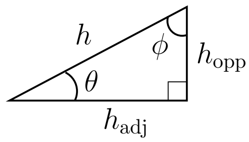
Phys 206 — Comprehensive Exam Formulae

Trigonometry and Vectors:

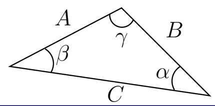
$$\sin 30^\circ = \cos 60^\circ = \tfrac{1}{2} \qquad \sin 36.9^\circ \approx \cos 53.1^\circ \approx \tfrac{3}{5}$$
$$\sin 45^\circ = \cos 45^\circ = \tfrac{1}{\sqrt{2}} \qquad \sin 53.1^\circ \approx \cos 36.9^\circ \approx \tfrac{4}{5}$$
$$\sin 60^\circ = \cos 30^\circ = \tfrac{\sqrt{3}}{2}$$

$$h_{\text{adj}} = h\cos\theta = h\sin\phi \qquad h^2 = h_{\text{adj}}^2 + h_{\text{opp}}^2$$
$$h_{\text{opp}} = h\sin\theta = h\cos\phi \qquad \tan\theta = \frac{h_{\text{opp}}}{h_{\text{adj}}}$$



Law of cosines: $C^2 = A^2 + B^2 - 2AB\cos\gamma$

Law of sines: $\dfrac{\sin\alpha}{A} = \dfrac{\sin\beta}{B} = \dfrac{\sin\gamma}{C}$



$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k} \qquad \hat{A} = \frac{\vec{A}}{|\vec{A}|}$$
$$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z = AB\cos\theta = A_\parallel B = AB_\parallel$$
$$\vec{A}\times\vec{B} = (A_yB_z - A_zB_y)\hat{\imath} + (A_zB_x - A_xB_z)\hat{\jmath} + (A_xB_y - A_yB_x)\hat{k}$$
$$|\vec{A}\times\vec{B}| = AB\sin\theta = A_\perp B = AB_\perp \quad \text{(direction via right-hand rule)}$$

Kinematics:

translational

$$\langle\vec{v}\rangle = \frac{\vec{r}_2-\vec{r}_1}{t_2-t_1} \qquad \vec{v} = \frac{d\vec{r}}{dt}$$
$$\langle\vec{a}\rangle = \frac{\vec{v}_2-\vec{v}_1}{t_2-t_1} \qquad \vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$$
$$\vec{r}(t) = \vec{r}_0 + \int_0^t \vec{v}(t')\,dt'$$
$$\vec{v}(t) = \vec{v}_0 + \int_0^t \vec{a}(t')\,dt'$$

rotational

$$\langle\omega\rangle = \frac{\theta_2-\theta_1}{t_2-t_1} \qquad \omega = \frac{d\theta}{dt}$$
$$\langle\alpha\rangle = \frac{\omega_2-\omega_1}{t_2-t_1} \qquad \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$
$$\theta(t) = \theta_0 + \int_0^t \omega(t')\,dt'$$
$$\omega(t) = \omega_0 + \int_0^t \alpha(t')\,dt'$$

— constant (linear/angular) acceleration only —

$$\vec{r}(t) = \vec{r}_0 + \vec{v}_0 t + \tfrac{1}{2}\vec{a}t^2 \qquad \theta(t) = \theta_0 + \omega_0 t + \tfrac{1}{2}\alpha t^2$$
$$\vec{v}(t) = \vec{v}_0 + \vec{a}t \qquad \omega(t) = \omega_0 + \alpha t$$
$$v_x^2 = v_{x,0}^2 + 2a_x(x-x_0) \qquad \omega_f^2 = \omega_0^2 + 2\alpha(\theta-\theta_0)$$

(and similarly for y and z)

$$\vec{r}(t) = \vec{r}_0 + \tfrac{1}{2}(\vec{v}_i + \vec{v}_f)t \qquad \theta(t) = \theta_0 + \tfrac{1}{2}(\omega_i + \omega_f)t$$

Energy and Momenta:

translational

$$K = \tfrac{1}{2}Mv^2$$
$$W = \int \vec{F}\cdot d\vec{r} \xrightarrow[\text{force}]{\text{const}} \vec{F}\cdot\Delta\vec{r}$$
$$P = \frac{dW}{dt} = \vec{F}\cdot\vec{v}$$
$$\vec{p}_{\text{cm}} = m_1\vec{v}_1 + m_2\vec{v}_2 + \ldots = M\vec{v}_{\text{cm}}$$
$$\vec{J} = \int \vec{F}dt = \Delta\vec{p}$$
$$\sum \vec{F}_{\text{ext}} = M\vec{a}_{\text{cm}} = \frac{d\vec{p}_{\text{cm}}}{dt}$$
$$\sum \vec{F}_{\text{int}} = 0$$

if $\sum F_{\text{ext},x} = 0,\ p_{\text{cm},x} = \text{const}$

rotational

$$\vec{\tau} = \vec{r}\times\vec{F} \text{ and } |\vec{\tau}| = r_\perp F = F_\perp r$$
$$K_{\text{rot}} = \tfrac{1}{2}I_{\text{tot}}\omega^2$$
$$W = \int \tau d\theta \xrightarrow[\text{torque}]{\text{const}} \tau\,\Delta\theta$$
$$P = \frac{dW}{dt} = \vec{\tau}\cdot\vec{\omega}$$
$$\vec{L} = \sum \vec{r}\times\vec{p} = I_1\vec{\omega}_1 + I_2\vec{\omega}_2 + \ldots = I_{\text{tot}}\vec{\omega}$$
$$\sum \vec{\tau}_{\text{ext}} = I_{\text{tot}}\vec{\alpha} = \frac{d\vec{L}}{dt}$$
$$\sum \vec{\tau}_{\text{int}} = 0$$

if $\sum \tau_{\text{ext},z} = 0,\ L_z = \text{const}$

— Work-energy and potential energy —

$$W = \Delta K \qquad W = -\Delta U \qquad E_{\text{tot},i} + W_{\text{other}} = E_{\text{tot},f}$$
$$U(r) = -\int \vec{F}\cdot d\vec{r}\ ; \quad U_{\text{grav}} = Mgy_{\text{cm}}\ ; \quad U_{\text{elas}} = \tfrac{1}{2}k\Delta x^2$$
$$F_x(x) = -dU(x)/dx \qquad \vec{F} = -\vec{\nabla}U = -\left[\frac{\partial U}{\partial x}\hat{\imath} + \frac{\partial U}{\partial y}\hat{\jmath} + \frac{\partial U}{\partial z}\hat{k}\right]$$

Quadratic:

$$ax^2 + bx + c = 0 \quad\Rightarrow\quad x_{1,2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

Derivatives:

$$\frac{d}{dt}(at^n) = nat^{n-1}$$
$$\frac{d}{dt}\sin at = a\cos at$$
$$\frac{d}{dt}\cos at = -a\sin at$$

Integrals:

if $f(t) = at^n$, then $\begin{cases} \int_{t_1}^{t_2} f(t)dt = \frac{a}{n+1}(t_2^{n+1} - t_1^{n+1}) \\ \int f(t)dt = \frac{a}{n+1}t^{n+1} + C \end{cases}$

$(n \neq -1)$

$$\int \sin at\,dt = \frac{-1}{a}\cos at$$
$$\int \cos at\,dt = \frac{1}{a}\sin at$$

Constants/Conversions:

g = 9.80 m/s^2 = 32.15 ft/s^2 (Earth, sea level)

$\approx$ 10 m/s^2 $\approx$ 33 ft/s^2

G = 6.674×10^{-11} N $\cdot$ m^2/kg^2 — 1 mi = 1609 m

1 lb = 4.448 N — 1 ft = 12 in

$\Leftrightarrow$ 0.454 kg (Earth, sea level) — 1 in = 2.54 cm

1 rev = 360° = 2π radians

Circular motion:

$$a_{\text{rad}} = \frac{v^2}{R} \qquad a_{\text{tan}} = \frac{d|\vec{v}|}{dt} = R\alpha$$
$$T = \frac{2\pi R}{v} \qquad s = R\theta \qquad v_{\text{tan}} = R\omega$$

Relative velocity:

$$\vec{v}_{A/C} = \vec{v}_{A/B} + \vec{v}_{B/C}$$
$$\vec{v}_{A/B} = -\vec{v}_{B/A}$$

Forces:

Newton's Law: $\sum\vec{F} = m\vec{a},\quad \vec{F}_{B\text{ on }A} = -\vec{F}_{A\text{ on }B}$

Hooke's Law: $F_x = -k\Delta x$

friction: $|\vec{f}_s| \le \mu_s|\vec{n}|,\quad |\vec{f}_k| = \mu_k|\vec{n}|$

Centre-of-mass:

$$\vec{r}_{\text{cm}} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \ldots + m_n\vec{r}_n}{m_1 + m_2 + \ldots + m_n}$$

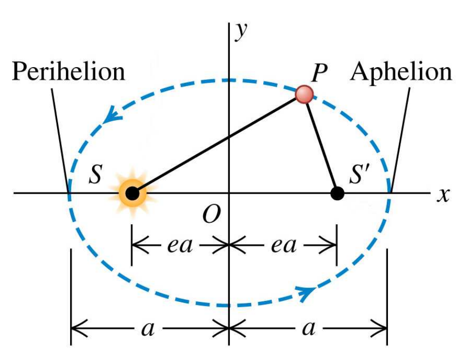
(and similarly for $\vec{v}$ and $\vec{a}$)

Gravity:

$$\vec{F}_{\text{grav}} = -G\frac{M_1M_2}{r^2}\hat{r} \qquad U_{\text{grav}} = -G\frac{M_1M_2}{r}$$

Kepler's Laws:

1st:

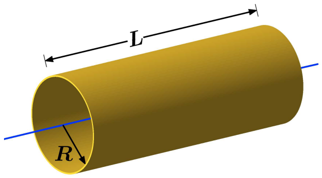


2nd: $\vec{r}\times\vec{v} = \text{constant}$

3rd: $T = \dfrac{2\pi a^{3/2}}{\sqrt{GM}}$

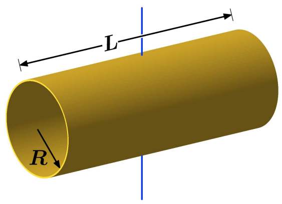
Moments of inertia:

Thin cylinder



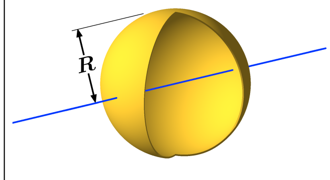
$I = MR^2$

Thin cylinder



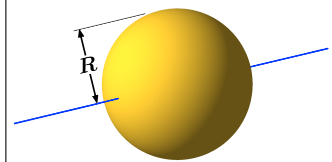
$I = \frac{1}{2}MR^2 + \frac{1}{12}ML^2$

Hollow sphere



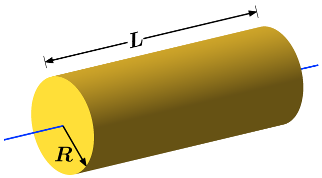
$I = \frac{2}{3}MR^2$

Solid sphere



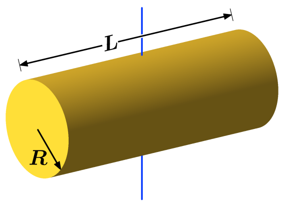
$I = \frac{2}{5}MR^2$

Solid cylinder



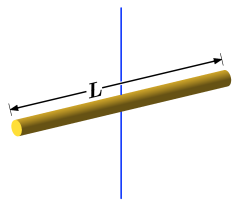
$I = \frac{1}{2}MR^2$

Solid cylinder



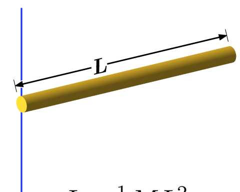
$I = \frac{1}{4}MR^2 + \frac{1}{12}ML^2$

Thin rod



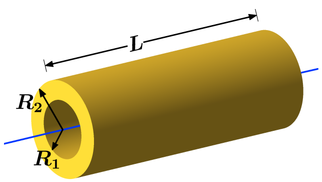
$I = \frac{1}{12}ML^2$

Thin rod



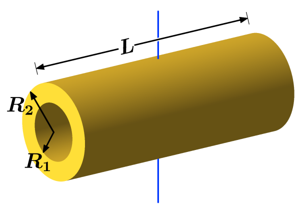
$I = \frac{1}{3}ML^2$

Thick cylinder



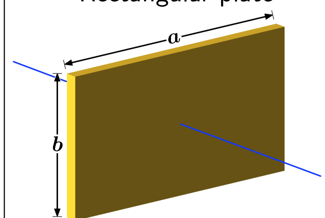
$I = \frac{1}{2}M(R_1^2 + R_2^2)$

Thick cylinder



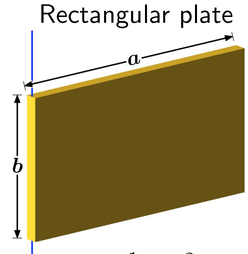
$I = \frac{1}{4}M(R_1^2 + R_2^2) + \frac{1}{12}ML^2$

Rectangular plate



$I = \frac{1}{12}M(a^2 + b^2)$

Rectangular plate



$I = \frac{1}{3}Ma^2$

- ⇝ For a point-like particle of mass M a distance R from the axis of rotation: $I = MR^2$
- ⇝ Parallel axis theorem: $I_p = I_{\text{cm}} + Md^2$
- ⇝ A disk is a cylinder of negligible length; the I for a disk may be found by setting $L = 0$ in the formulae for cylinders

Periodic motion:

$$\omega = 2\pi f = 2\pi/T$$

pendulum: $T = 2\pi\sqrt{L/g} = 2\pi\sqrt{I_P/mgd}$

spring: $T = 2\pi\sqrt{m/k}$

torsion: $T = 2\pi\sqrt{I/\kappa}$

Simple harmonic motion:

$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

$\Leftrightarrow \quad a(t) = -\omega^2 x(t)$

or $\quad \alpha(t) = -\omega^2\theta(t)$

$$x(t) = A\cos(\omega t + \phi_0)$$
$$v(t) = -\omega A\sin(\omega t + \phi_0)$$
$$a(t) = -\omega^2 A\cos(\omega t + \phi_0)$$

$$\tan\phi_0 = \frac{-v_0}{\omega x_0}$$

$$A^2 = x_0^2 + \left(\frac{v_0}{\omega}\right)^2$$