Chapter 9 - Rotation of Rigid Bodies

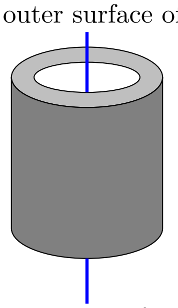
Physics 206

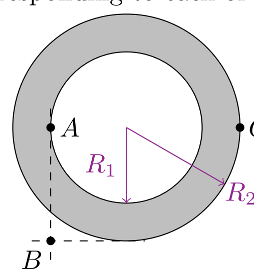
For any problems where you are given a variable/symbol and a value for that variable, make sure to solve the problem symbolically first. Your final answer should then only contain the variables that you are given values for in the problem, constants that appear on the equation sheet and numbers like 2 or π.

Group 1 Problems:

Problem 1: It has been argued that power plants should make use of off-peak hours to generate mechanical energy and store it until it is needed during peak load times. One suggestion has been to store the energy in large flywheels spinning on nearly frictionless ball bearings. Consider a flywheel made of iron ($\rho = 7800$ kg/m^3) in the shape of a 10.0 cm thick uniform disk.
(a) What would the diameter of such a disk need to be if it is to store 10.0 megajoules of kinetic energy when spinning at 90.0 rpm about an axis perpendicular to the disk at its center?
(b) What would be the centripetal acceleration of a point on its rim when spinning at this rate?

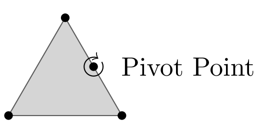
Problem 2: Consider the thin-walled hollow cylinder shown below which has a moment of inertia about its center of mass $I_{CM} = \frac{1}{2}M(R_1^2 + R_2^2)$. On the right side of the figure, three different axes of rotation are shown, all parallel to the axis through the center-of-mass shown on the left: A is on the inner radius, B is to the left of center by R_1 and below the center by R_2, and C is on the outer surface of the cylinder. Find moment of inertia corresponding to each of the axes of rotation.



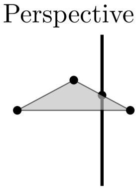


Problem 3: What is the moment of inertia of a system of three identical point particles with masses m located at the vertices of an equilateral triangle with sides $2\sqrt{3}\ell$ for the rotation axis perpendicular to the plane of the triangle and passing through the middle of one of the sides?

Top View



Perspective View



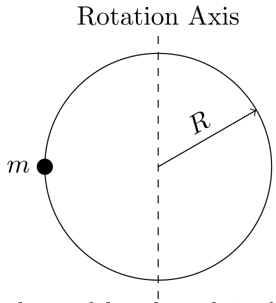
Problem 4: Suppose a roulette wheel is spinning at 1.00 rev/s.
a) How long will it take for the wheel to come to rest if it experiences an angular acceleration of -0.0200 rad/s^2?
b) How many rotations will it complete in that time?
c) If the radius of the wheel is 40.0 cm, what is the magnitude of acceleration of a point on the rim after it has completed 3 full rotations?

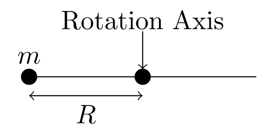
Problem 5: The angular acceleration of a rotating object is $\alpha(t) = 1.60t - 2.40$. At $t = 2.00$ s the angular velocity is -30.0 rad/s. What is the angular displacement between $t = 0.00$ and $t = 5.00$ s?

Group 2 Problems:

Problem 6: A computer disk is turned on starting from rest and has a constant angular acceleration. It takes $\tau = 0.340$ s to complete its *second* complete revolution.
a) How long did it take to complete the first revolution?
b) What is the angular acceleration?

Problem 7: A uniform, solid disk with mass m and Radius R is pivoted about a horizontal axis through its center. A small object of the same mass m is glued to the rim of the disk. If the disk is released from rest with the small object at the end of a horizontal radius, find the angular speed when the small object is directly below the axis.



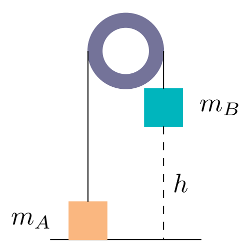


Problem 8: A slender rod has length is ℓ and has mass M. A small m_1 solid sphere is welded to one end of the rod, and a small m_2 solid sphere is welded to the other end. The rod is able to pivoting about a stationary, frictionless axis at its center. If the rod is held horizontal and then released from rest:
a) What is the linear speed of the m_2 sphere as it passes through its lowest point?
b) Repeat a) but treat the rotation axis as the location of m_1.

Problem 9: A thin, light wire is wrapped around the rim of a solid cylinder, with a small block suspended from the free end of the wire. The block, with a mass of 12.0 kg, is released from rest and falls, causing the uniform 10.0 kg cylinder of diameter 30.0 cm to turn about a frictionless axle through its center. How far will the mass have to descend to give the cylinder 480 J of kinetic energy?

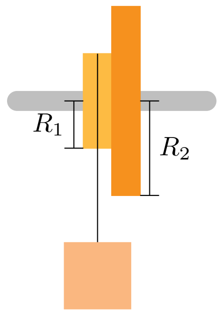
Problem 10: A solid sphere and a hollow sphere are simultaneously released from rest at the top of an incline that makes an angle of 30 degrees above the horizontal. The release point is 2.00 m above the bottom of the hill. Assume both spheres roll without slipping down the hill, have a mass of 1.75 kg and a radius of 20.0 cm.
a) What percentage of each object's total kinetic energy is rotational? Is this position dependent?
b) By what percentage would you change the mass of the hollow sphere so that the two would have the same speed when they reach the bottom? What if you changed the radius instead?

Problem 11: Referring back to the Atwood's Machine problem from earlier this semester. Assume that $m_A = 8.00$ kg, $m_B = 18.0$ kg and $h = 1.25$ m. The pulley has inner radius of 7.50 cm and outer radius of 10.0 cm. If the pulley is massless (ideal) then m_B hits the ground with a speed that you can calculate. What does the mass of the real pulley need to be so that it hits the ground at 60% of this speed?



Group 3 Problems:
Problem 12: A thin, light wire is wrapped around the rim of a wheel. The wheel rotates about a stationary horizontal axle that passes through the center of the wheel. The wheel has radius R and moment of inertia for rotation about the axle of I. A small block with mass m is suspended from the free end of the wire. When the system is released from rest, the block descends with constant acceleration. The bearings in the wheel at the axle are rusty, so friction there does W of work as the block descends a distance H. What is the magnitude of the angular velocity of the wheel after the block has descended H?

Problem 13: Two metal disks, one with radius R_1 and mass M_1 and the other with radius R_2 and mass M_2 are welded together and mounted on a frictionless axle through their common center (see the figure below).
a) What is the total moment of inertia of the two disks?
b) A light string is wrapped around the edge of the smaller disk, and a block with mass m is suspended from the free end of the string. If the block is released from rest at a distance H above the floor, what is its speed just before it strikes the floor?
c) Repeat part b) assuming the string is wrapped around the edge of the larger disk. Which case has the larger speed and why?



Problem 14: What is the first time after 3:00 pm that the minute hand and hour hand are at the same position on a clock? What is the second time? Round your answers to the nearest second.
When working through and typing up the solution to this problem for the online assignment, use the subscript m to indicate the minute hand and h to indicate the hour hand. For example, $\theta_{0,h}$ would be the initial angle of the hour hand and $\omega_m(t)$ would be the angular velocity of the minute hand at time t. Be sure to write a sentence or two explaining why you set the values for each variable to the values you chose. Once you set up your initial equations, write a sentence or two indicating what physics and mathematical principles/laws you are using to reach your final answer. Once your reach your answer for the first time after 3:00, write a sentence explaining why your answer falls within some expected limits. After that, explain your reasoning for why you modified your starting equations in the way you did to solve for the second time. Again, when you reach your final answer, does it fall within some expected limits?

Problem 15: An object with length ℓ and is centered on the origin has a linear mass density $\lambda(x) = \lambda_0 + \lambda_1 x^2$ where λ_0 and λ_1 are both positive constants.
a) Integrate to find the moment of inertia of this object about the z-axis.
b) Integrate to find the moment of inertia of this object about an axis that is parallel to the z-axis and through the left edge.
c) What do you notice about the difference in the answers to a) and b)?