Chapter 6 - Work and Kinetic Energy

Physics 206

For any problems where you are given a variable/symbol and a value for that variable, make sure to solve the problem symbolically first. Your final answer should then only contain the variables that you are given values for in the problem, constants that appear on the equation sheet and numbers like 2 or π.

Group 1 Problems:

Problem 1: A box with mass m is given an initial velocity v_0 and is slid along a rough table. If the box slides a distance D before coming to rest, what is the coefficient of kinetic friction?

Problem 2: A block is sliding across the ground at 5.00 m/s. There is a coefficient of kinetic friction of 0.300. How far does the box slide before coming to rest? (Solve this using Work-Energy theorem.)

Problem 3: A variable force $F(x) = ax^3 - bx^2 + c$ is applied to an object from x_1 to x_2. The mass of the object is m and the object is at rest at x_1 what is the velocity at x_2? Would Δv be the same if the velocity at x_1 was not zero?

Problem 4: Consider a constant force $\vec{F} = 4\hat{\imath} - 3\hat{\jmath} + 2\hat{k}$. What is the work done by this force on a particle as it moves from $\vec{r} = 3\hat{k}$ to $\vec{r} = 2\hat{\imath}$? What is the angle between the force vector and the displacement vector?

Problem 5: An object with a mass $m = 2.00$ kg has a velocity vector $\vec{v}_1 = 3\hat{\imath} - 2\hat{\jmath} + 4\hat{k}$. Some external force acts on the object and causes the velocity to change to $\vec{v}_2 = -3\hat{\imath} + 5\hat{\jmath} + 2\hat{k}$. How much work did this force do on the object?

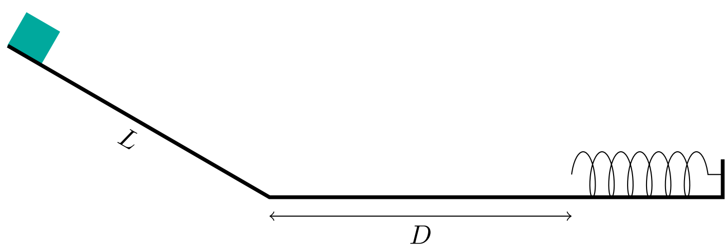
Group 2 Problems:

Problem 6: A box with mass m starts at rest and slides down a ramp of length L that is θ above the horizontal. At the end of the ramp, the box slides along a horizontal surface for a distance D before it makes contact with an uncompressed spring with force constant k.

(a) If friction can be ignored everywhere, what is the maximum distance the spring is compressed?

(b) If the friction coefficients of the ramp are μ_s and μ_k, and the coefficients for the horizontal surface are $\mu_s/2$ and $\mu_k/3$, what would the initial speed of the box need to be in order to compress the spring by the same amount. For this, ignore friction once the box becomes in contact with the spring.

(c) Repeat (b) but do not ignore friction once in contact with the spring. This should be a group 3 problem on its own.

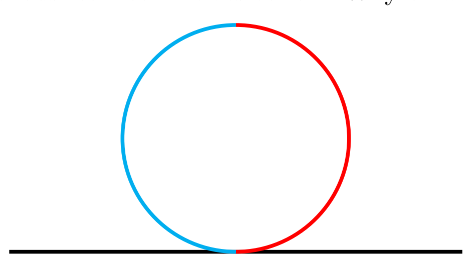


Problem 7: A neighborhood daredevil is trying to sled through a complete loop of radius R. The daredevil and sled have a combined mass m. Since they put their loop together through whatever materials they could find, the loop has two different materials. The first part of the loop (shown in red on the right) is a rough material, and the second part of the loop (shown in blue on the left) is a slick material. The daredevil has a velocity v_0 when they reach the rough material and start the loop.

(a) What is the maximum work done by friction so that they still clear the loop?

(b) What is the speed of the daredevil when they complete the loop and are back to the bottom?

(c) Assuming you were given the values for R, m and v_0, is it possible to find a formula for the coefficient of friction between the sled and the rough material using methods learned in class so far? Why or why not?



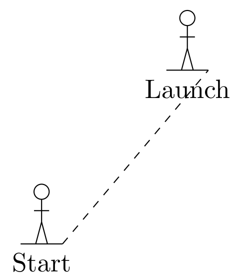
Problem 8: A rocket of mass m is attempting to land on Planet X which has acceleration due to gravity of g_X. The rocket activates a thruster when they are a distance h above the ground. The engine outputs a thrust $F_{\text{thrust}}(y) = ay^3 - by + c$ as

a function of height where $y = 0$ is the ground. What is the maximum speed the rocket can be moving when the thruster is activated so that the velocity when it hits the ground is exactly zero?

Problem 9: The safety system to protect drivers going down hills during an ice storm consists of a bumper, which can be considered a stiff spring, at the bottom of the hill. In the scenario you are given, the car, mass m, starts from rest at the top of a hill which makes an angle θ with the horizontal. The distance that the car slides from the top of the hill until it is stopped by the spring is L. For the worst case scenario, assume that there is no frictional force between the car and road due to the ice. If the maximum compression of the spring from its equilibrium position is D, your job is to calculate the required spring constant k in terms of m, D, L and θ.

Group 3 Problems:

Problem 10: During a summer weekend at a lake, you decide to try out a new attraction. This is effectively just a flat platform that a person stands on and it launches them (with constant acceleration) into the air and then into the lake. The platform accelerates a person from rest at an angle 40 degrees from the vertical. If a 65.0 kg person is taken from rest to 12.0 m/s in 0.800 s, what is the power applied by the normal, gravitational and frictional forces on the person?



Problem 11: Congratulations, you are now part of the USA Olympic Curling Team! Your job is to sweep in front of the stone (which has diameter D) to reduce the friction on the ice, thus allowing it to slide further. Your teammate slides the rock with an initial velocity v_0. The entire lane of ice has a coefficient of friction μ_k. After t seconds, you can begin to brush the ice, reducing the coefficient of friction (immediately after you start brushing) by 25%. The center of the bullseye is L meters away from the center of the stone when it is released.
(a) If the diameter of the bullseye is twice the diameter of the stone, and once you start to brush you continue until the stone stops, what is the range of distances you can brush so that the stone lands completely inside the bullseye?
(b) What if you were to brush this same distance over the start of the path and then stop brushing? Does the stone still land in the bullseye? Does this change the travel time of the stone? If so, does it increase or decrease?

Problem 12: A proton has a mass of $m_p = 1.67 \times 10^{-27}$ kg and is given an initial speed of $v = 3.00 \times 10^5$ m/s aimed directly at a uranium nucleus that is 5.00 m away. The proton is repelled by the uranium nucleus with a force that has the following form:

$$F(x) = \frac{\alpha}{x^2}$$

where x is the distance between the proton and the nucleus and $\alpha = 2.12 \times 10^{-26}$. For all parts of this problem assume the uranium nucleus stays at rest. At some point, the proton will briefly come to rest before being forced away from the nucleus. At what distance does this occur? What do the units of α have to be for the force equation to be dimensionally consistent?

Problem 13: Using the definition of work, the definition of power and the fundamental theorem of calculus, prove that $P = \vec{F} \cdot \vec{v}$. (Hint: it is useful to remember that position is a function of time and that $d\vec{\ell}$ is parallel to the trajectory which would imply it is in the same direction as the velocity.)