Chapter 3 - Motion in 2 and 3 Dimensions

Physics 206

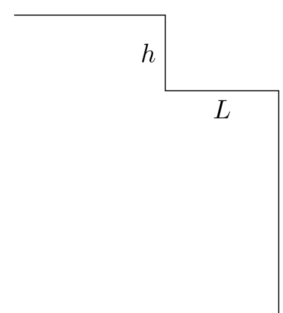
For any problems where you are given a variable/symbol and a value for that variable, make sure to solve the problem symbolically first. Your final answer should then only contain the variables that you are given values for in the problem, constants that appear on the equation sheet and numbers like 2 or π.

Group 1 Problems:

Problem 1: A water balloon is thrown horizontally with a speed of $v_0 = 5.00$ m/s from the roof of a building that is $h = 80.0$ m above the ground. At the same instant the balloon is released, a second balloon is thrown downward with the same speed $v_0 = 5.00$ m/s from the same height.
(a) Solve for the time difference between when the balloons hit the ground.
(b) Solve for the difference in the speeds of the balloons when they hit the ground.

Problem 2: A particle is traveling in the xy-plane with a velocity given by $v(t) = (3t^2 - 4t + 6)\hat{\imath} + (-7t - 3)\hat{\jmath}$. At $t = 0$ the particle is at $\vec{r} = -2\hat{\imath} + 3\hat{\jmath}$.
(a) What is the acceleration of the particle at time t=4 s?
(b) What is the velocity of the particle at time t=4 s?
(c) What is the position of the particle at time t=4 s?
(d) What is the magnitude of the average velocity over the interval of 3 to 5 s?
(e) What is the direction of the average acceleration over the interval of 3 to 5 s?

Problem 3: A small boat is capable of moving $v_b = 12.0$ mph in still water. It is in the middle of a wide river that is moving $v_r = 3.00$ mph. How much more time does it take for this boat to travel 2.75 miles upstream than 2.25 miles downstream?



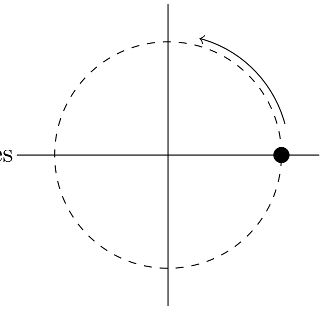
Problem 4: Because of your knowledge of physics, you have been hired as a consultant for a new spy movie, "Oldfinger". In one scene, the protagonist jumps horizontally off the top of a cliff to escape a villain. To make the stunt more dramatic, the cliff has a horizontal ledge a distance h beneath the top of the cliff which extends a distance L from the vertical face of the cliff. The stunt coordinator wants you to determine the minimum horizontal speed, in terms of L and h, with which they must jump so that they misses the ledge.

Problem 5: Two friends are running in a field. Bob is running with a velocity vector relative to the ground of $\vec{v}_1 = 5.00\hat{\jmath}$. Charlie is running with a vector relative to the ground that has a magnitude $v_2 = 4.00$ m/s and is in a direction θ away from Bob's. What is the velocity vector of Charlie relative to Bob when θ is:
(a) 30, 45, 60, 90, 120, 135, 150 and 180 degrees?
(b) What patterns or symmetries do you notice?

Problem 6: An object is moving along a horizontal path in uniform circular motion. The radius of the motion is 0.850 m and it completes a full loop in 0.400 seconds. The object is moving counterclockwise and at $t = 0$ is at $\theta = 0$ as shown below.

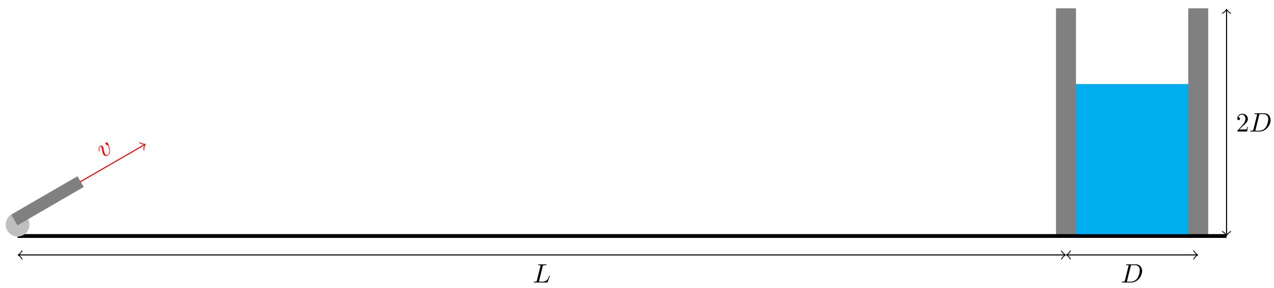
(a) What is the velocity vector when the object is at $\theta = 90.0$ degrees?
(b) What is the velocity vector when the object is at $\theta = \pi$ radians?
(c) What is the magnitude of the average acceleration vector between $t = 0$ and $t = 0.200$ (when it is at $\theta = 180$ degrees)?
(d) What is the average acceleration vector between when the object is at $\theta = 270$ degrees and when it is at $\theta = 360$ degrees. Assume this is during the same loop.

$\theta = 90$ degrees

$\theta = 180$ degrees  $\theta = 0$ degrees

$\theta = 270$ degrees

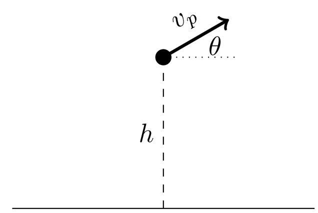
Group 2 Problems:
Problem 7: A local high school is hosting a science fair and you decide to build a cannon to launch a ball into a water tank. The cannon fires the ball with a velocity v at an angle of $\theta = 30$ degrees. The water tank has a diameter of D with walls of height $2D$.



a) How far back from the front wall of the tank, L, should the cannon be placed so that the ball just clears the front top edge of the tank?
b) Assuming the initial speed is $v = 2\sqrt{gD}$, can the cannon actually land a ball in the tank? Justify your answer with words using logic and/or a mathematical equation.

Problem 8: An airplane is flying with a speed v_p at an angle θ relative to the horizontal. When the plane is h above the ground it releases a box. Neglecting any effects due to air resistance, calculate the following.

(a) How much time does it take for the box to hit the ground.
(b) What maximum height does the box reach?
(c) What is the horizontal displacement the box travels from the time it was released to the time it first strikes the ground?



Problem 9: The end of a propeller with a radius of 1.50 m has a ***speed*** which can be given by the formula $v(t) = -2.5 + 3t^2$ when $t > \sqrt{2.5/3}$. What is the magnitude of the acceleration of the end of the propeller at $t = 1.50$ s?

Problem 10: A ballista is a siege weapon that can fire bolts at around 50.0 m/s (112 mph). Assume that there is a tower and the leader of the location you are laying siege to is in a room with a window 20.0 m above the ground and that your ballista is on a platform 5.00 m above the ground. You are trying to fire your ballista at an angle of 35.0 degrees above the horizontal so that it flies into the window (naturally for the sole reason of scaring said leader, not to harm him). Are there any distances from the tower that the platform can be in order for your bolt to go through the window? If so, what are they?

Problem 11: A small boat is capable of moving 12 mph in still water. It is on the bank of a 0.75 mile wide river and the water is moving 3 mph. There is a dock on the other side of the river. The boat aims at that dock, starts the engine and then does not change trajectory. How far does the boat miss the dock if the dock is:
(a) Directly across the river.
(b) 2.00 miles upstream.
(c) 2.00 miles downstream.
(d) How much time does it take the boat to reach the other side in all three cases?

Group 3 Problems:
Problem 12: A team of engineering students enter a competition to build a tennis ball launching robot. The balls will be launched from ground level down a hallway with a height $h = 13$ ft.
(a) Assume the robot can launch the balls with a fixed velocity $v = 15.0$ m/s but can vary the launch angle to anything between horizontal and vertical. What is the maximum distance the ball can travel without hitting the ceiling?
(b) Assume instead the robot could launch at a fixed angle $\theta = 28.0$ degrees but vary the velocity. What is the maximum distance in this case?

Problem 13: Student-engineers decided to test a rocket they had built on a horizontal segment of highway. The rocket burns its fuel very quickly and was launched like a projectile at an angle of $\phi = 45$ degrees above the horizontal. At the time of the launch another student is driving their truck at a constant velocity v_T along this same highway in the same direction as the rocket. The rocket reaches a maximum height of h and lands in the back of the truck as it comes back to the ground. Assume that back of the truck and launching point of the rocket are at the exactly same height. Neglect air resistance.

(a) What distance from the launching point was the truck when the rocket landed in it?
(b) Where was the truck, relative to the launch point of the rocket, when it was launched?
(c) An observer in the back of the truck measures the velocity and acceleration of the rocket while in flight. In the reference frame of the truck, what is the velocity and acceleration of the rocket when it is at its highest point on its trajectory?

Problem 14: A toy rocket is launched with an initial velocity of $v_0 = 12.0$ m/s in the horizontal direction from the roof of a $h = 30.0$ m tall building. The rocket's engines produce a constant horizontal acceleration of $a(t) = 1.6t$ (in SI units) and does not produce an acceleration in the vertical direction. What horizontal distance does the rocket travel by the time it first hits the ground? What do the units of the 1.6 have to be for the acceleration to be in SI units?

Problem 15: Where does the boat in **Problem 11** have to aim in order to hit the dock. Do this for all three cases. Also calculate the time to cross the river and the speed the boat is moving relative to a stationary observer on the shore in all three cases.
Note for this problem: Solving this for the case where the goal is to end directly across the river is a reasonably straightforward case to solve symbolically. For the arbitrary/general case, this is more difficult. You should get to a point where you have something like $C = A\sin\theta + B\cos\theta$. You may want to consult an online resource for an analytical solution to this.