Chapter 13 - Universal Gravitation

Physics 206

For any problems where you are given a variable/symbol and a value for that variable, make sure to solve the problem symbolically first. Your final answer should then only contain the variables that you are given values for in the problem, constants that appear on the equation sheet and numbers like 2 or π.

Group 1 Problems:
Problem 1: Comets travel around the sun in elliptical orbits with large eccentricities. If a comet has speed v_1 when at a largest distance of r_1 from the center of the sun, what is its speed when at a closest distance of r_2?

Problem 2: While preparing its budget for the next fiscal year, NASA wants to report to the nation a rough estimate of the cost (per kilogram) of launching a modern satellite into near-Earth orbit. Taking into account that mass of the Earth is 6.0×10^{24} kg and radius of the Earth is 6400 km.
(a) Determine the energy in KW-h, necessary to place a 1.0 kg object in low-Earth orbit. In low-Earth orbit, the height of the object above the surface of Earth is much smaller than Earth's radius. Take the orbital height to be 300 km.
(b) If this energy can be obtained at a typical electrical energy rate of \$0.15/KW-h, what is the minimum cost of launching a 400-kg satellite into low-Earth orbit? Neglect any effects due to air resistance.

Problem 3: One of the brightest comets of the 20th century was Comet Hyakutake, which passed close to the sun in early 1996. The orbital period of this comet is estimated to be about 30,000 years. Find the semi-major axis of this comet's orbit. Assume the mass of the sun is 2.00×10^{30} kg. Compare it to the average sun-Pluto distance (5.91×10^9 km) and to the distance to Alpha Centauri, the nearest star to the sun, which is 4.3 light-years distant.

Problem 4: A satellite with mass m is in a circular orbit around the Earth at a distance r from the center of the Earth. How much energy is required to move this satellite to a circular orbit with twice the radius? Let the mass of the earth be M_E.

Problem 5: The Rosetta spacecraft landed a probe on the comet Churyumov-Gerasimenko. The comet has a mass of (approximately) 1.00×10^{13} kg and has an average density of (approximately) 400 kg/m^3. It is far from spherical, but go ahead and assume it to be a sphere. Estimate g at its surface.

Group 2 Problems:
Problem 6: A landing craft with mass M is in a circular orbit h above the surface of a planet. The period of the orbit is T. The astronauts in the lander measure the diameter of the planet to be D. The lander sets down at the north pole of the planet.
(a) What is the weight of an astronaut with mass m as he steps out onto the planet's surface?
(b) What is the apparent weight of the astronaut if he landed at the equator of the planet instead and if one day on that planet took a time T_d?

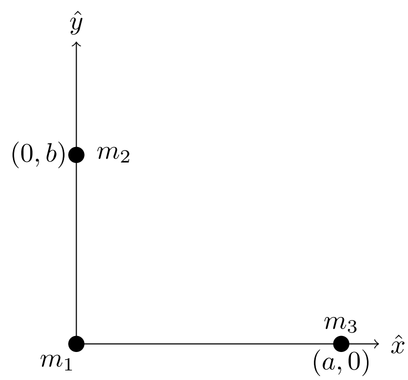
Problem 7: An unmanned spacecraft is in a circular orbit around the Moon (mass M_M, radius R_M), observing the lunar surface from an altitude of H. To the dismay of scientists on earth, an electrical fault causes an on-board thruster to fire, decreasing the speed of the spacecraft by δv. If nothing is done to correct its orbit, with what speed will the spacecraft crash into the lunar surface? Treat δv as a very small change.

Problem 8: Two point masses M_1 and M_2 are separated by a distance d. A third mass m_3 is a distance x away from M_1 and between the masses. Assume $M_1 > M_2$.
(a) What is the magnitude of the force acting on m_3?
(b) What does x have to be so that the net force on m_3 is exactly zero?

Problem 9: Suppose a satellite is in an elliptical orbit around the Earth and has a speed of 4.460 km/s at perigee (the closest to the Earth) and 2.263 km/s at apogee (the farthest from the Earth). If the distance from the surface of the Earth to the satellite at perigee is 20,200 km, what is the distance from the surface of the Earth to apogee? The mass of the Earth is 5.972×10^{24} kg. Assume the radius of the Earth is 6371 km.

Problem 10: Planet X has 80 times the earth's mass and five times its radius. It has a circular orbit with a period of 24 years around a star that has three times the sun's mass. (For the purposes of this problem assume the Earth has a circular orbit)
Part (a): Let us define that the distance from the Earth to the sun is 1 AU (astronomical unit). What is the Planet X's orbital radius in AU?
Part (b): An astronaut has a weight on Earth of 125 lbs. What is her weight on the surface of Planet X?

Group 3 Problems:
Problem 11: A system contains 3 masses, m_1, m_2 and m_3 at the locations given in the figure below.
(a) What is the magnitude of the net gravitational force acting on m_1?
(b) What is the total potential energy of this system in this orientation?
(c) How much work would it take to move m_2 which is at rest at its current position to a position infinitely far away, also at rest?
(d) Now that m_2 is infinitely far away, m_3 starts to orbit around m_1 (which stays in place) in a circular orbit. What is the magnitude of the angular momentum of m_3?



Problem 12: Repeat **Problem 8a** in order to find the net force vector acting on m_3 so that it is correct no matter whether x is in between the masses or outside the masses. It helps to draw a picture here.

Problem 13: In **Problem 9**, define the displacement vector between the Earth and the satellite to be $\vec{r}$. If the distance between the surface of the earth and the satellite is 33,100 km, what is the angle between $\vec{r}$ and the velocity vector of the satellite?